Alban City School
Calculation Policy

Rationale

This policy outlines a model progression through written strategies for addition, subtraction, multiplication and division in line with the maths curriculum at Alban City School. Through the policy, we aim to link key manipulatives and visual representations in order that the children can be accelerated through each strand of calculation. We know that school wide policies, such as this, can ensure consistency of approach, enabling children to progress stage by stage through models and representations they recognise from previous teaching, allowing for deeper conceptual understanding and fluency. As children move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to the children's level of understanding. However, it is expected that the majority of children in each class will be working at age-expected levels as set out in the school's curriculum.

The importance of mental mathematics

While this policy focuses on written calculations in mathematics, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The following checklists outline the key skills and number facts that children are expected to develop throughout the school.

To add and subtract successfully, children should be able to:

- recall all addition pairs to 9 + 9 and number bonds to 10, 20 and 100
- recognise addition and subtraction as inverse operations
- add mentally a series of one digit numbers (e.g. 5 + 8 + 4)
- add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. 600 + 700, 160 — 70)
- partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into 70 + 4 or 60 + 14)
- use estimation by rounding to check answers are reasonable

To multiply and divide successfully, children should be able to:

- add and subtract accurately and efficiently
- recall multiplication facts to 12 x 12 = 144 as well as the associated division facts e.g. 144 ÷ 12 = 12
- Use multiplication and division facts to estimate how many times one number divides into another etc.
- know the outcome of multiplying by 0 and by 1 and of dividing by 1
- understand the effect of multiplying and dividing whole and decimal numbers by 10, 100 and 1000
- recognise factor pairs of numbers (e.g. that 15 = 3 x 5, or that 40 = 10 x 4) and increasingly able to recognise common factors
- notice and recall with increasing fluency inverse facts
- partition numbers into 100s, 10s and 1s or multiple groupings
- understand how the principles of commutative, associative and distributive laws apply or do not apply to multiplication and division
- understand the effects of scaling by whole numbers and decimal numbers or fractions
- understand correspondence where n objects are related to m objects
- investigate and learn rules for divisibility

Further support

Each week the children complete a CLIC test as part of their maths work. This provides children with a regular opportunity to revisit key calculation methods even if they are not part of the current maths unit. The children are then supported 1:1 or in a small group to address any misconceptions they may still hold. In Key Stage 2, we also deliver weekly arithmetic sessions to support knowledge across the four operations, including with fractions.

EYFS – Revised Framework (2021)

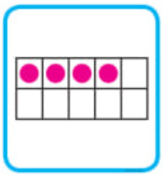
The Early Years Foundation Stage Framework has been revised and the amendments take effect from September 2021.

1. **Deepen understanding of numbers up to 10**
    - Understand the relationship between these numbers
    - Find patterns with numbers to 10
    - Use manipulatives (objects to help understand a mathematical concept e.g. Numicon)
    - Organise numbers using a 10s frame

2. **Subitising up to 5**

Subitising involves recognising numbers in different forms and recognising how many there are without counting them.

Examples



3. **Automatically recall number bonds to 5 and some to 10, including double facts**

Examples

0 + 5 = 5   1 + 4 = 5   2 + 3 = 5   3 + 2 = 5   4 + 1 = 5   5 + 0 = 5

4. **Compare quantities to 10, including identifying one more or one less than a number.**

5. **The composition of numbers to 10.**

Examples

- 6 is made up of 5 and 1.
- 6, 7, 8 and 9 lie between 5 and 10 on a number line.
- Numbers that can be made out of groups of 2 are even.
- The numbers can be partitioned in different ways e.g. 4 and 3. If we know one part, then we can find the other part.

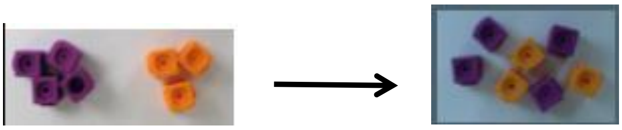
These fundamental skills will underpin the following learning in addition, subtraction, multiplication and division.

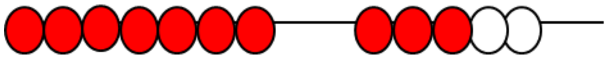
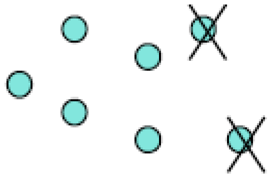
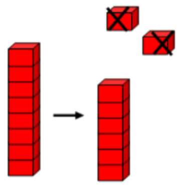
Progression in addition and subtraction
EYFS – Year 6

Addition and subtraction are connected.

Part	Part
Whole	

Addition names the whole in terms of the parts and **subtraction** names a missing part of the whole.

<table>
<tr><th>Addition</th><th>Subtraction</th></tr>
<tr><td>
EYFS/Y1
<u>Combining two sets (aggregation)</u>
Putting together – two or more amounts or numbers are put together to make a total
4 + 3 = 7

Count one set, then the other set. Combine the sets and count again. Starting at 1.

Counting along the bead bar, count out the 2 sets, then draw them together, count again. Starting at 1.

</td><td>
EYFS/Y1
<u>Taking away (separation model)</u>
Where one quantity is taken away from another (or crossed out) to calculate what is left.
7 – 2 = 5

Multilink towers - to physically take away objects.

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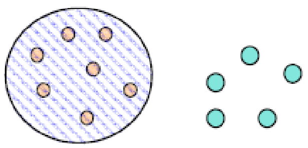
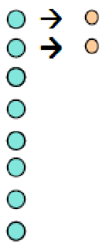
<tr><td>
<u>Combining two sets (augmentation)</u>
This stage is essential in starting children to calculate rather than counting
Where one quantity is increased by some amount. Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number.

<u>Counters:</u>

7 + 5 = 12
Start with 7, then count on 8, 9, 10, 11, 12
</td><td>
<u>Finding the difference (comparison model)</u>
Two quantities are compared to find the difference.

8 – 2 = 6
<u>Counters:</u>

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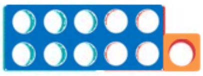
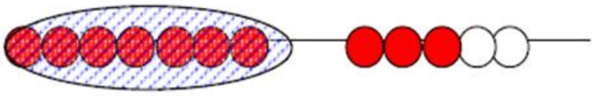
Numicon



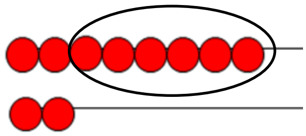
Start with 10 and count on 1

Bead strings:



Make a set of 7 and a set of 5. Then count on from 7.

Bead strings:

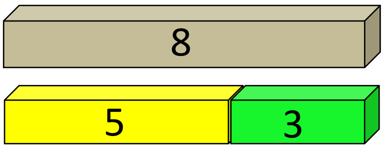


Make a set of 8 and a set of 2. Then count the gap.

Multilink Towers:

5 + 3 = 8

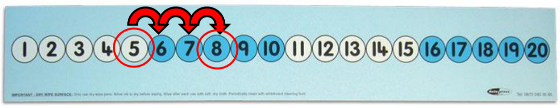
Cuisenaire Rods:



5 + 3 is the same as 8

8 is the same as 3 + 8

Number tracks



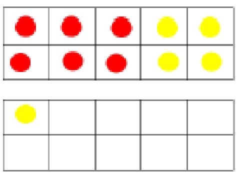
Start on 5 then count on 3 more.

Cubes or Numicon may be used alongside a number line

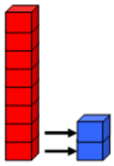
Regrouping to make 10

Regrouping to make 10 by using ten frames and counters/cubes or using numicon:

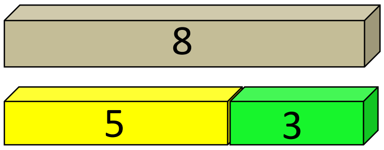
6 + 5



Multilink Towers:



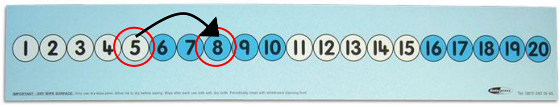
Cuisenaire Rods:



8 – 5 = 3

8 – 3 = 5

Number tracks:



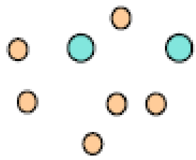
Start with the **smaller** number and count the *gap* to the **larger** number.

1 set within another (part-whole model)

The quantity in the whole set and one part are known, and may be used to find out how many are in the unknown part.

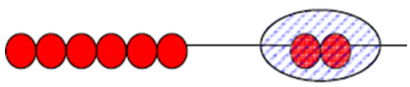
8 – 2 = 6

Counters:



Bead strings:

8 – 2 = 6

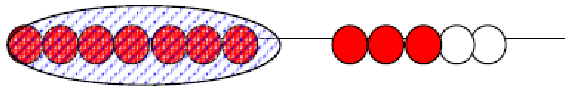


Bridging through 10s

This stage encourages children to become more efficient and begin to use known facts.

Years 1 and 2 onwards

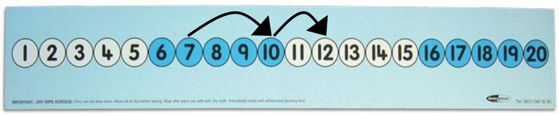
Bead string:



7 + 5 is decomposed / partitioned into 7 + 3 + 2.

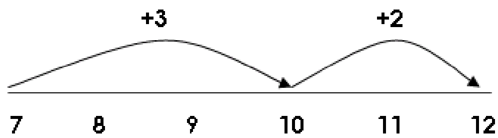
The bead string illustrates **'How many more to the next multiple of 10?'** (*Children should link their knowledge of number bonds*) and then **'If we have used 3 of the 5 to get to 10, how many more do we need to add on?'**

Number track:

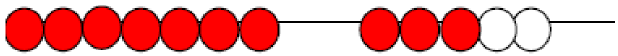


Steps can be recorded on a number track prior to transition to number line.

Number line



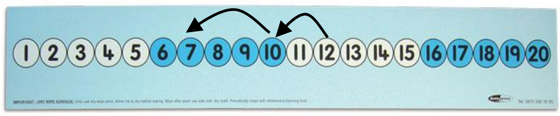
Bead string:



12 – 7 is decomposed / partitioned in 12 – 2 – 5.

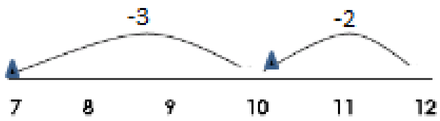
The bead string illustrates **'From 12, how many to the last/previous multiple of 10?'** and then **'If we have used 2 of the 7 we need to subtract, how many more do we need to count back?'**

Number Track:



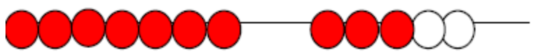
Steps can be recorded on a number track prior to transition to number line.

Number Line:

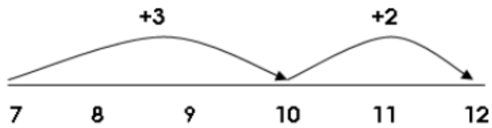


Counting up method

Bead string:



12 – 7 becomes 7 + 3 + 2.
Starting from 7 on the bead string **'How many more to the next multiple of 10?'** (Children should apply number bonds), '**How many more to get to 12?**'.

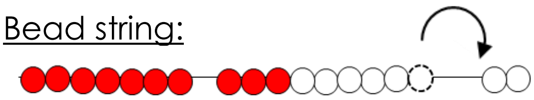


Compensation model (adding 9 and 11) (optional)

Encourages efficiency and application of known facts (use ten)

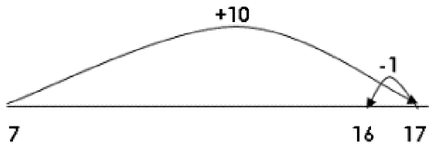
7 + 9

Bead string:



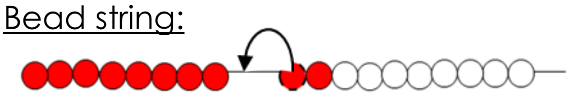
Children find 7, then add on 10 and then adjust by removing 1.

Number line:



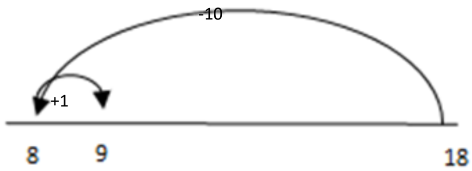
18 – 9

Bead string:



Children find 18, then subtract 10 and then adjust by adding 1.

Number line:



Working with larger numbers
Tens and ones + tens and ones

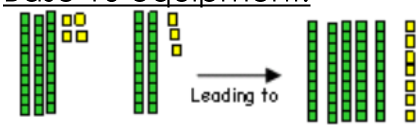
Children should transition onto Base 10 equipment and understand the abstract nature of the single 'tens' sticks and 'hundreds' blocks

Year 2

Partitioning (Aggregation model)

34 + 23 = 57

Base 10 equipment:

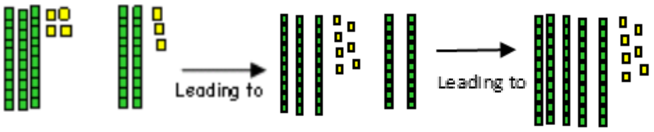


Create the two sets with Base 10 equipment and then combine; ones with ones, tens with tens.

Partitioning (Augmentation model)

Base 10 equipment:

Children begin counting from the first set of ones and tens and not counting from 1. Begin with the ones in preparation for formal columnar method.



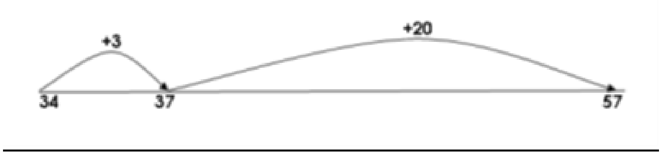
Take away (Separation model)

57 – 23 = 34

Base 10 equipment:

Children remove the lower quantity from the larger set, starting with the ones and then the tens. In preparation for formal decomposition.

Number line:



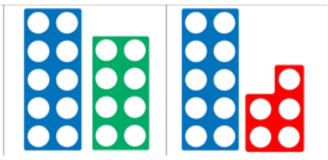
Children can begin to use an informal method to support, record and explain their method **(optional)**.

30 + 4 + 20 + 3

50 7

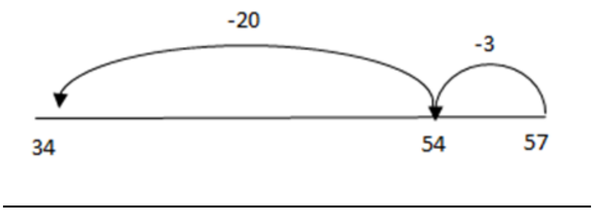
57

Additional resources: Numicon can be used to partition numbers



10 8 10 5

Number Line:



Children can began to use an informal method to support, record and explain their method **(optional)**.

(50 + 7) - (20 + 3)

30 4

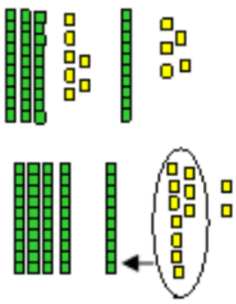
34

Bridging with larger numbers

Once secure in partitioning for addition, children begin to explore exchanging.
*Children consider: '**What happens if the ones are greater than 10?**'*
Introduce the term 'exchange'. Using Base 10 equipment, children exchange ten ones for a single tens rod, which is equivalent to crossing the tens boundary on the bead string or number line.

Base 10 equipment:

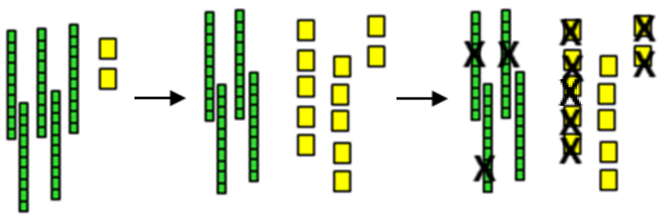
37 + 15 = 52



Discuss always counting on from the larger number, irrespective of the order of the calculation when working with addition.

Base 10 equipment:

52 – 37 = 15



In the diagram above, 5 tens and 2 ones (50 + 2) are changed to 4 tens and 12 ones (40 + 12) both of which still represent 52. Three tens and seven ones can then be removed.

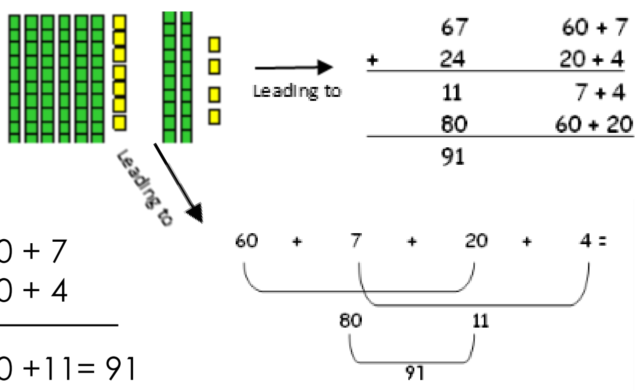
Expanded Vertical Method

Children are introduced to the expanded vertical method to make the link between Base 10 equipment, partitioning and recording using the expanded vertical method.

Year 3 and Year 4

Base 10 equipment:

67 + 24 = 91

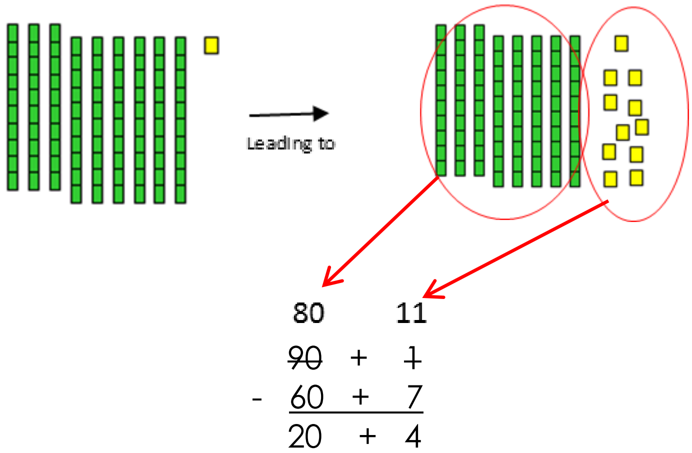


60 + 7
20 + 4

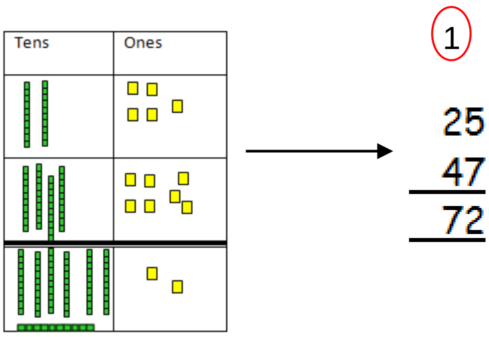
80 + 11 = 91

Base 10 equipment:

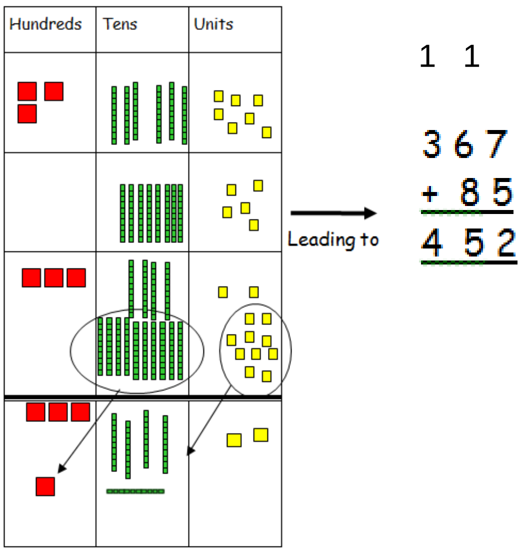
91 – 67 = 24



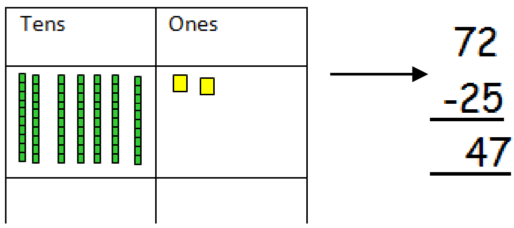
Compact method



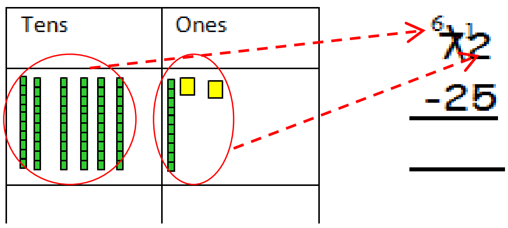
10 ones are changed for 1 ten which is moved to the tens column. This is represented above the calculation.

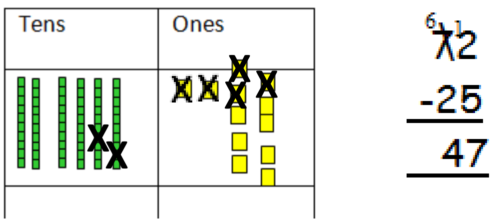


Compact decomposition



7 tens and 2 ones are changed for 6 tens and 12 ones. This is represented in the calculation.





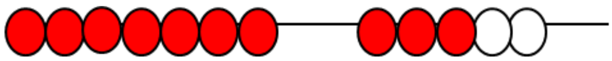
Vertical acceleration

By returning to earlier manipulative experiences, children are supported to make links across mathematics, encouraging 'If I know this…then I also know…' thinking.

Decimals

Ensure that children are confident in counting forwards and backwards in decimals – using bead strings to support.

Bead strings:



Each bead represents 0.1, each different block of colour equal to 1.0

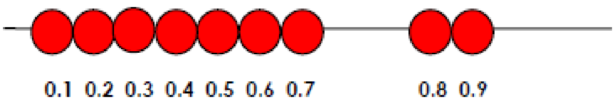
Base 10 equipment:

0.1 1.0 10.0

Addition of decimals

Aggregation model of addition

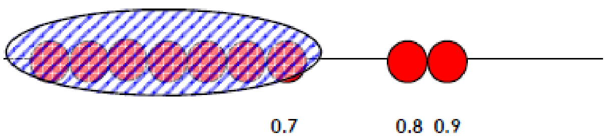
Counting both sets – starting at zero.
0.7 + 0.2 = 0.9



Augmentation model of addition

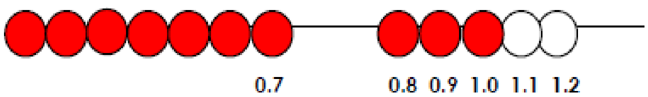
Starting from the first set total, count on to the end of the second set.

0.7 + 0.2 = 0.9



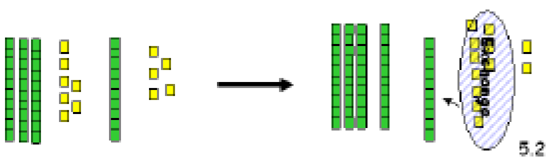
Bridging through 1.0

Encouraging connections with number bonds.
0.7 + 0.5 = 1.2 (add on 0.3 before adding on 0.2)



Partitioning

3.7 + 1.5 = 5.2

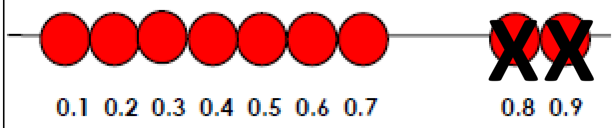


Represent both values using base 10 before combining.

Subtraction of decimals

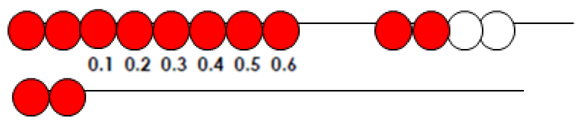
Take away model

0.9 – 0.2 = 0.7



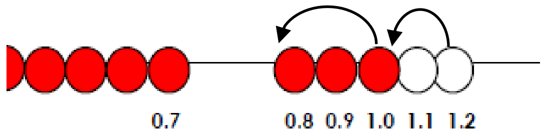
Finding the difference (or comparison model):

0.8 – 0.2 =



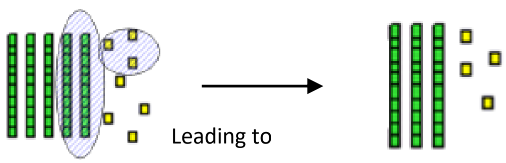
Bridging through 1.0

Encourage efficient partitioning.
1.2 – 0.5 = 1.2 – 0.2 – 0.3 = 0.7



Partitioning

5.7 – 2.3 = 3.4



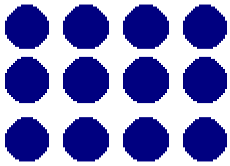
Represent both values using base 10 before removing.

Gradation of difficulty- addition:	**Gradation of difficulty- subtraction:**
1. **3 + 4 =** *(Numbers less than 10)* 2. **1 + 3 + 2 =** *(Number less than 10)* 3. **7 + 5 =** *(Answers crossing a 10 boundary)* 4. **4 + 3 + 6 =** *(Answers crossing a 10 boundary)* 5. **35 + 68 =** *(Answers crossing a 100 boundary)* 6. **417 + 825 =** *(Answers crossing a 1000 boundary and so on)* 7. **3154 + 257 =** *(2 numbers with a different number of digits)* 8. **2.15 + 3.46 =** *(Adding decimal numbers up to 2 decimal places with the same number of decimal places)* 9. **1.2 + 3.78 =** *(Adding 2 or more decimal numbers with a different number of decimal places)*	1. **5 – 3 =** *(Numbers less than 10)* 2. **63 – 31 =** **543 – 210 =** *(No exchange)* 3. **51 – 27 =** *(Exchanging tens for ones)* 4. **216 – 154 =** *(Exchanging hundreds for tens)* 5. **514 – 286 =** *(Exchanging hundreds to tens and tens to ones)* 6. **2381 – 1658 =** *(Same as step 5 but with a different number of digits)* 7. **6.53 – 3.26 =** *(Decimals up to 2 decimal places with the same number of decimal places)* 8. **4.1 – 1.76 =** *(Subtract 2 or more decimal numbers with a range of decimal places)*

Progression in Multiplication and Division

Multiplication and division are connected.
Both express the relationship between a number of equal parts and the whole.

Part	Part	Part	Part
Whole			



The following array, consisting of four columns and three rows, could be used to represent the number sentences:

3 x 4 = 12   4 x 3 =12   3 + 3 + 3 + 3 = 12   4 + 4 + 4 =12.

And it is also a model for division:

12 ÷ 4 = 3   12 ÷ 3 = 4   12 – 4 – 4 – 4 = 0   12 – 3 – 3 – 3 – 3 =

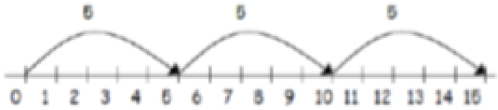
Multiplication	Division
EYFS/Year 1 **Early experiences** Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem solving activities involving equal sets or groups. Repeated grouping/repeated addition (using different resources)    	**EYFS/Year 1** Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s. 

Herts for Learning

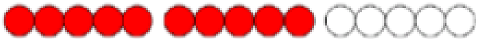
Year 1 onwards

Repeated addition (repeated aggregation)

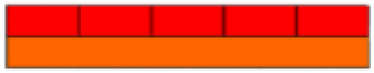
3 times 5 is 5 + 5 + 5 = 15 **OR** 5 lots of 3 **OR** 5 x 3. Children learn repeated addition can be shown on a number line.



It can also be shown on a bead string.



Children also learn to partition totals into equal trains using Cuisenaire Rods

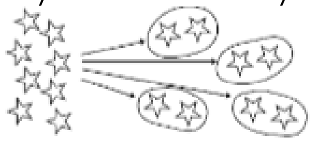


5 x 3 = 15

Year 1 onwards

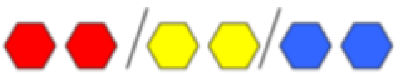
Sharing equally

8 sweets are shared between 4 people. How many sweets do they each get?



Grouping or repeated subtraction

There are 6 sweets. How many people can have 2 sweets each?

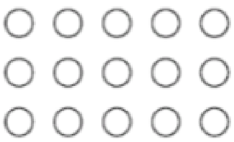


Some children may progress onto repeated subtraction using a number line. **(See below)**

Year 1 onwards

Arrays

Children learn to model a multiplication calculation using an array. This model supports their understanding of **commutativity** and the development of the grid in a written method. It also supports the finding of factors of a number in later years.

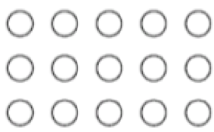


5 x 3 = 15

3 x 5 = 15

Year 1 onwards

Children learn to model a division calculation using an array. This model supports their later understanding of the development of partitioning and the 'bus stop method' in a written method. This model also connects division to **finding fractions** of discrete quantities in later years.



15 ÷ 3 = 5

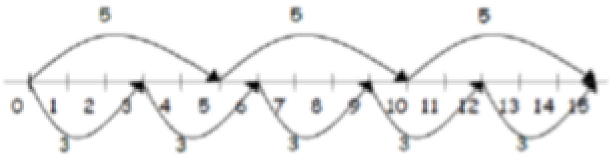
15 ÷ 5 = 3

Year 2 onwards

Commutativity

Children learn 3 x 5 has the same total as 5 x 3. This can be shown on the number line.

3 x 5 = 15 is the same as 5 x 3 = 15

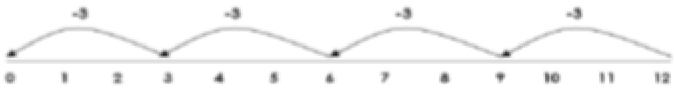


Year 2 onwards

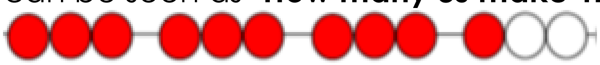
Children learn that division is **not** commutative and link this to subtraction.

Repeated subtraction using a bead string or number line

12 ÷ 3 = 4 (Moving back in jumps of 3)



The bead string helps children understand division calculations, recognising that 12 ÷ 3 can be seen as '**How many 3s make 12?**'



Year 3 onwards

Scaling

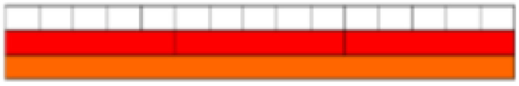
This is an extension of augmentation used in addition. With multiplication, the quantity is increased by a scale factor.

Example:

You have 3 giant marbles and swap each one for 5 of your friend's small marbles, you will end up with 15 marbles.

This can be written as:

1 + 1 + 1 = 3 □ scaled up by 5 □ 5 + 5 + 5 = 15



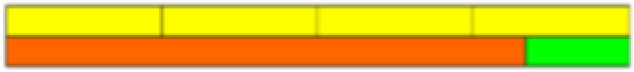
Example:

Find a ribbon that is 4 times as long as the blue ribbon.

Children should be aware that if they multiply by a number less than 1, this scaling reduces the size of the quantity.

Example: Scaling 3 by a factor of 0.5 would reduce it to 1.5 (0.5 x 3 = 1.5)

Cuisenaire Rods also help children to interpret division calculations.

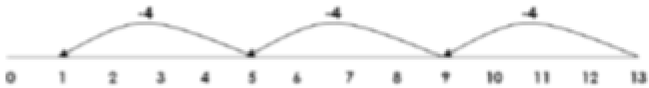


6 ÷ 2 =

Grouping involving remainders

Children move onto calculations involving remainders using a number line/bead string.

13 ÷ 4 = 3 r1



Year 2 onwards

Inverse operations

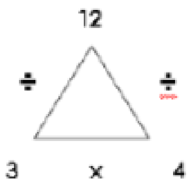
Trios can be used to model the 4 related multiplication and division facts. Children learn to state the 4 related facts.

3 x 4 = 12

4 x 3 = 12

12 ÷ 3 = 4

12 ÷ 4 = 3



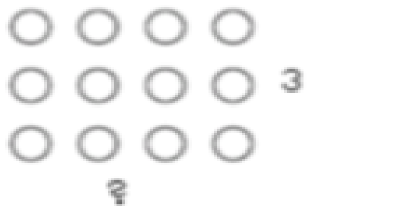
Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

□ x 5 = 20 3 x Δ = 18 O x □ = 32

24 ÷ 2 = □ 15 ÷ O = 3 Δ ÷ 10 = 8

This can also be supported using arrays: e.g.

3 X ? = 12

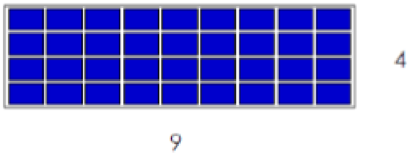


Year 3 onwards

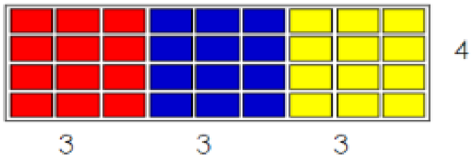
Partitioning for multiplication

Arrays are useful to help children visualise how to partition larger numbers into more useful representation.

9 x 4 = 36



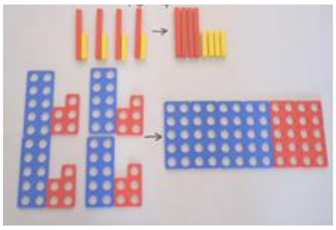
Children may be flexible with how they use number and can break the array into more manageable chunks.



9 x 4 = (3 x 4) + (3 x 4) + (3 x 4) = 12 + 12 + 12

Use numicon, base 10 and cuisenaire rods

4 X 15 =

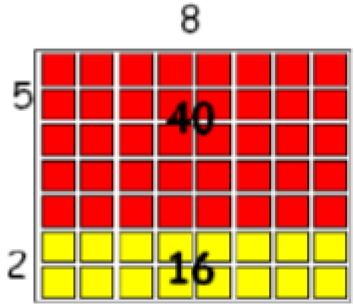


Year 3 onwards

Partitioning for division

The array is also a flexible model for division of larger numbers

56 ÷ 8 = 7



Children could break this down into more manageable arrays, as well as using their understanding of the inverse relationship between division and multiplication.

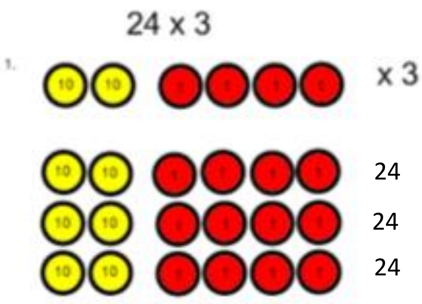
56 ÷ 8 = (40 ÷ 8) + (16 ÷ 8) = 5 + 2 = 7

Year 3 onwards

Arrays leading into the grid method

Children continue to use arrays and partitioning, where appropriate, to prepare them for the grid method of multiplication. Arrays can be represented as 'grids' in a shorthand version and by using place value counters to show multiples of ten, hundred etc.

e.g. 24 x 3 =

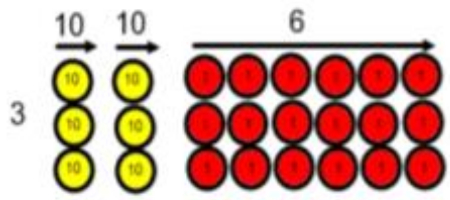


Year 3 onwards

Arrays leading into chunking and then long and short division

Children continue to use arrays and partitioning where appropriate, to prepare them for the 'chunking' and short method of division. Arrays are represented as 'grids' as a shorthand version.

e.g. 78 ÷ 3 =

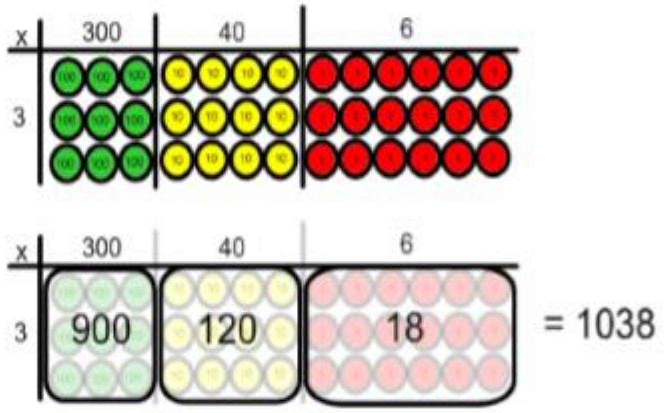


78 ÷ 3 = (30 ÷ 3) + (30 ÷ 3) + (18 ÷ 3) =
10 + 10 + 6 = 26

Year 4 onwards

Grid method

This written strategy is introduced for the multiplication of TO x O to begin with. It may require column addition methods to calculate the total. **Place value counters may still be used.**



Year 4 onwards

The vertical method- 'chunking' leading to long division

See above for example of how this can be modelled as an array using place value counters.

78 ÷ 3 =

```
   78
-  30     (10 x 3)
   48
-  30     (10 x 3)
   18
-  18      (6 x 3)
    0
```

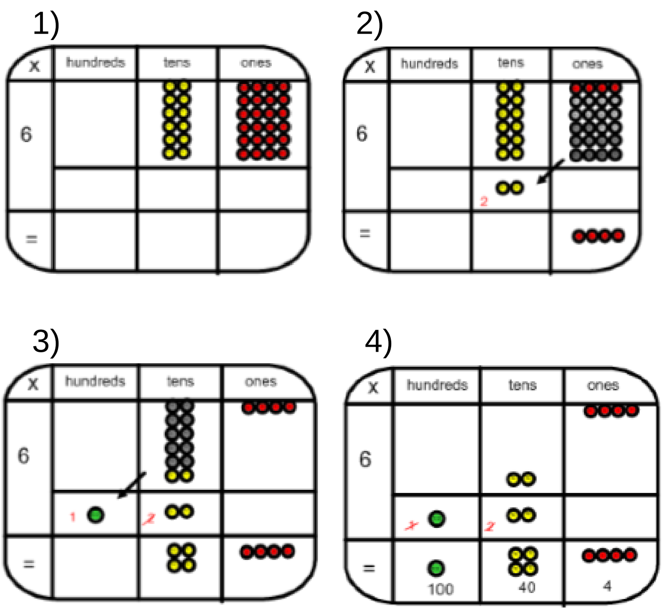
So 78 ÷ 3 = 10 + 10 + 6 =26

Year 5 onwards

Short multiplication — multiplying by a single digit

The array using place value counters becomes the basis for understanding short multiplication first without exchange before moving onto exchanging. **Example**: 24 x 6

```
   24
x   6
  144
  1 2
```



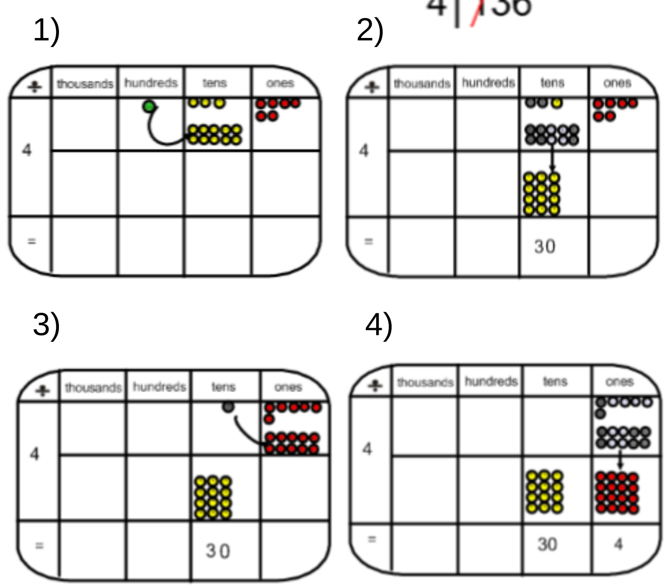
Year 5 onwards

Short division — dividing by a single digit

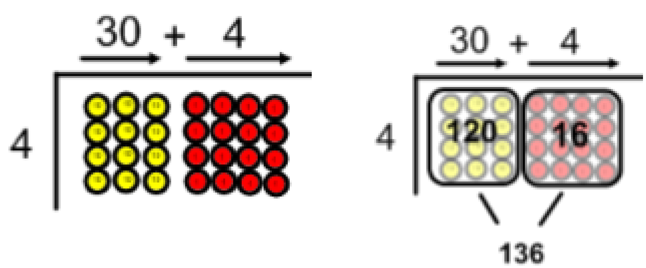
Whereas we can begin to group counters into an array to show short division working

Example: 136 ÷ 4

```
    34
4 | 136
```



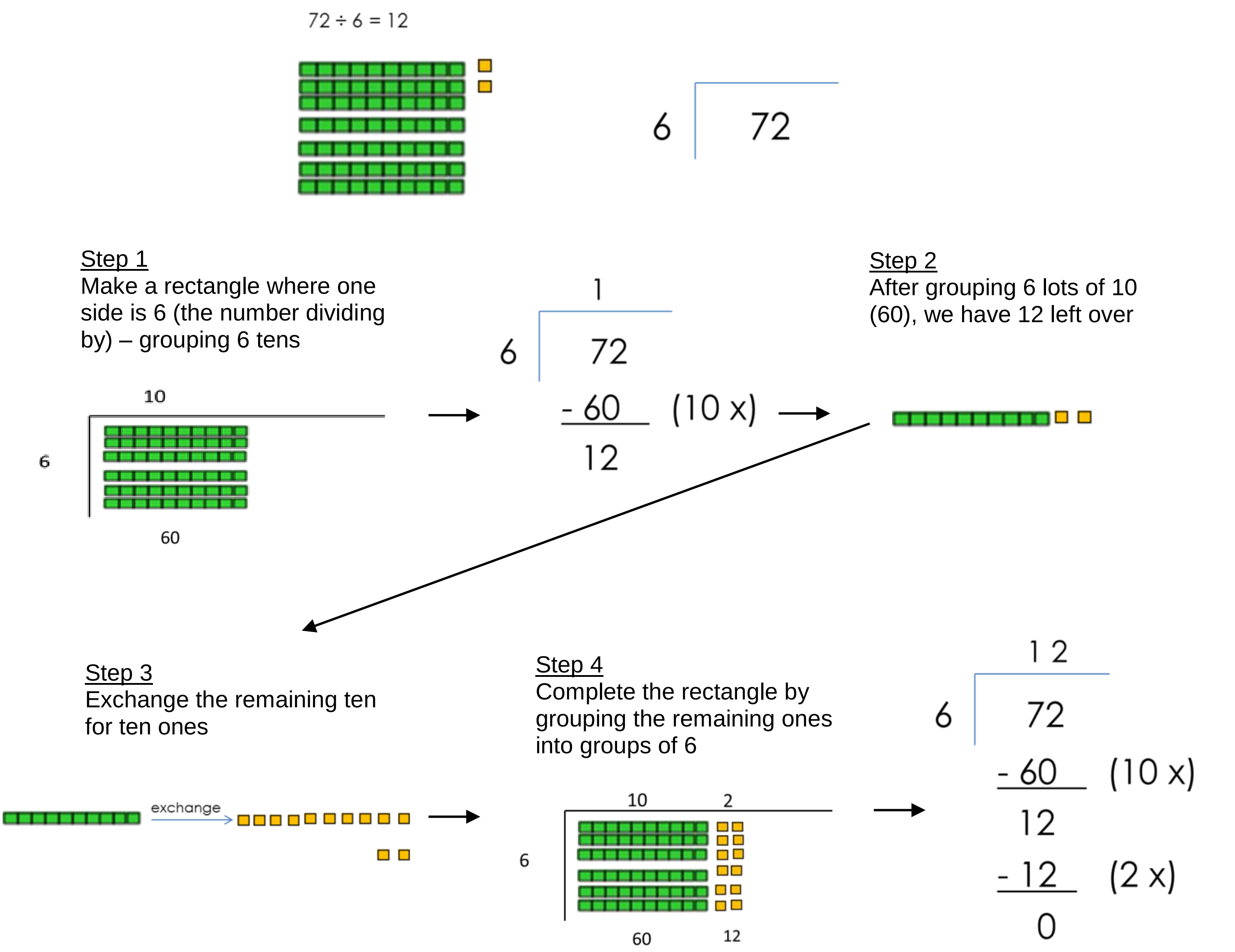
Shown below in the bus stop format:



<u>Long multiplication - multiplying by more than one digit</u>	**<u>Long division - dividing by more than one digit</u>**
Children will refer back to grid method by using place value counters or Base 10 equipment. Initially there will be no exchange and using modelling of writing recording. Move onto TO x TO, HTO x TO etc. ***Children will eventually move from concrete to pictorial and finally the abstract form.***	Children should be reminded about partitioning numbers into multiples of 10, 100 etc. before recording as either:- 1. Chunking model of long division using Base 10 equipment 2. Sharing model of long division using place value counters See the following 3 pages for exemplification of these methods. ***Children will eventually move from concrete to pictorial and finally the abstract form.***

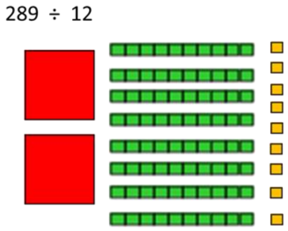
<u>Chunking model of long division using Base 10 equipment</u>

Divisors less than 10: *This model links strongly to the array representation; so for the calculation 72 ÷ 6 = ? - one side of the array is unknown and by arranging the Base 10 equipment to make the array we can discover this unknown. The written method should be written alongside the equipment so that children make links.*



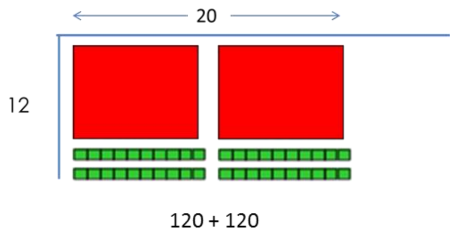
Herts for Learning

Divisors between 11 and 19: *Children may benefit from practice to make multiples of tens using the hundreds and tens and tens and ones.*



12 | 289

Step 1
Make a rectangle where one side is 12 (the number dividing by) using hundreds and tens



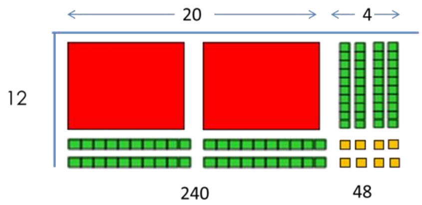
→

```
      2
12 | 289
   - 240   (20 x )
      49
```

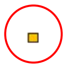
→

Step 2
After grouping 12 lots of 20 (240), we have 49 left over

Step 3
Make groups of 12s using tens and ones



→



Step 4
No more groups of 12 can be made and 1 remains

→

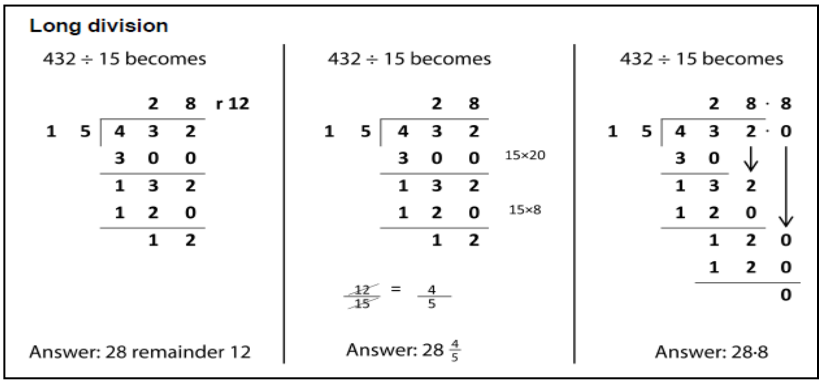
```
     24 r1
12 | 289
   - 240   (20 x )
      49
   -  48   (4 x )
       1
```

Dealing with remainders

Remainders should be given as integers (whole numbers), but children need to be able to decide what to do after division, such as rounding up or down accordingly.
e.g.:

- I have 62p. How many 8p sweets can I buy?
- Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed?

Gradation of difficulty for expressing remainders

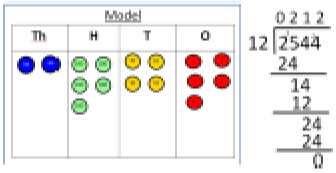


1) Whole number remainder

2) Remainder expressed as a fraction of the divisor *(including simplification)*

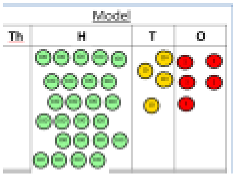
3) Remainder expressed as a decimal

The following models demonstrates the movement from concrete to pictorial and then to abstract for long division.

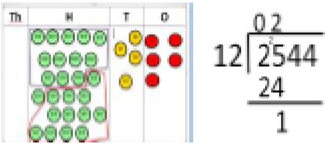


2544 ÷ 12

How many groups of 12 thousands do we have? None

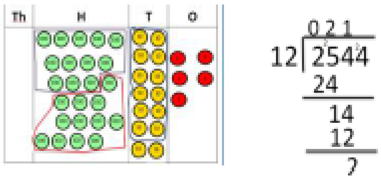


Exchange 2 thousand for 20 hundreds.

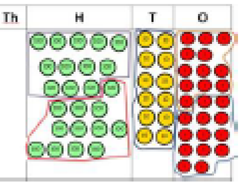


How many groups of 12 are in 25 hundreds? 2 groups. Circle them.

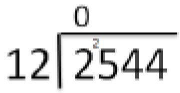
We have grouped 24 hundreds so can take them off and we are left with one.



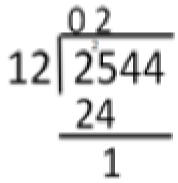
Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.



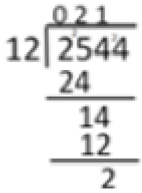
Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2



Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds.

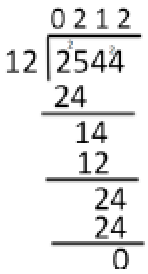


Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.



Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens?

The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left.



Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.

Gradation of difficulty (short multiplication)

1. **12 x 4 =** *(TO x O with no exchange)*
2. **13 x 6 =** *(TO x O with exchange of ones into tens)*
3. **213 x 2 =** *(HTO x O with no exchange)*
4. **204 x 4 =** *(HTO x O with exchange of ones into tens)*
5. **241 x 3 =** *(HTO x O with exchange of tens into hundreds)*
6. **174 x 5 =** *(HTO x O with exchange of ones into tens and tens into hundreds)*
7. *Same as steps 3-6 but with greater number digits x O e.g.* ***1391 x 5 =***
8. **2.1 x 4 =** *(O.t x O with no exchange)*
9. **1.4 x 6 =** *(O.t x O with exchange of tenths to ones)*
10. *Same as steps 8 - 9 but with greater number of digits and may include a range of decimal places x O e.g.* ***2.54 x 3 =***

Gradation of difficulty (short division)

1. **64 ÷ 2 =** *(TO ÷ O with no exchange and no remainder)*
2. **87 ÷ 4 =** *(TO ÷ O with no exchange with but includes a remainder)*
3. **52 ÷ 4 =** *(TO ÷ O with exchange but no remainder)*
4. **98 ÷ 7 =** *(TO ÷ O with exchange and with remainder)*
5. **816 ÷ 4 =** *(Zero in the answer e.g. 204)*
6. *Same as steps 1-5 but now using HTO ÷ O e.g.* ***264 ÷ 2 =***
7. *Same as steps 1-5 but with a greater number of digits ÷ O e.g.* ***2852 ÷ 4 =***
8. *Same as steps 1-5 but with a decimal dividend e.g.* ***7.5 ÷ 5 =*** ***0.12 ÷ 3 =***
9. **4284 ÷ 12 =** *(Dividing by a 2 digit number)*

See page 16 for gradation of difficulty with displaying remainders